Mathematical model

The full Navier-Stokes equations, the equation of energy for a non-perfect gas, and a two-parameter equation of state are included into the mathematical model. Two-scale splitting of the pressure is used. To close the set of equations, the integral mass balance is involved. In dimensionless form the governing equations can be written as follows:

$$\frac{\partial\rho}{\partial t}+\nabla\left(\rho\vec{U}\right)=0$$

$$\rho\frac{\partial U}{\partial t}+\rho(\vec{U}\nabla)\vec{U}=-\nabla p+\frac{1}{\mathrm{Re}}\left(2\nabla\left(\eta\dot{D}\right)-\nabla\left(\frac{2}{3}\eta-\varsigma\right)\nabla\vec{U}\right)+\frac{Ra}{\mathrm{Pr}\,\Theta\,\mathrm{Re}^2}\rho\,\vec{g}$$

$$\rho\frac{\partial T}{\partial t}+\rho\,(U\nabla)T=-(\gamma_0-1)T\left(\frac{\partial P}{\partial T}\right)_\rho\nabla\vec{U}+\frac{\gamma_0}{\mathrm{Re}\,\mathrm{Pr}}\nabla(\lambda\nabla T)+$$

$$+\frac{\gamma_0(\gamma_0-1)M^2}{\mathrm{Re}}\left(2\eta\dot{D}^2-\nabla\left(\frac{2}{3}\eta-\varsigma\right)\left(\nabla\vec{U}\right)^2\right)$$

$$P=P(\rho,T)$$

$$P=\langle P\rangle+\gamma_0M^2p$$

$$\int_V p\,dv=0$$

Here, ρ, $\vec{U}$, $\dot{D}$, and T are the density, velocity, strain rate tensor, and temperature; P, $\langle P\rangle$ and p are the total, mean and dynamic pressure; $\vec{g}$ is the body force acceleration; η, ς, and λ are the dynamic and bulk viscosity and the thermal conductivity; dv is a volume element, and V is the total volume.

The stratification in state of equilibrium (for example, initially) is described by the linear approximation ($\vec{r}=(x,y)$ is the radius-vector; superscript «*» denotes boundary values):

$$\rho=\rho^*\left(1+\left(\frac{\partial\rho^*}{\partial P^*}\right)_{T^*}\varepsilon_g\vec{g}(\vec{r}-\vec{r}^*)\right),\quad P=P^*+\rho^*\varepsilon_g\vec{g}(\vec{r}-\vec{r}^*)$$

The total pressure P is decomposed into two parts (the mean $\langle P\rangle$ and dynamic p components) and the parts are nondimensionalized using the different scales.

The governing dimensionless equations include the following parameters (the Reynolds, Rayleigh, Prandtl, Mach numbers, the temperature difference, and the ratio of specific heats for a perfect gas)

$$\mathrm{Re}=\frac{\rho_c'U'l'}{\eta_0'},\quad Ra=\frac{\Theta'g'l'^3\rho'^2(c_{v0}'+B')}{T'_c\lambda_0'\eta_0'},\quad \mathrm{Pr}=\frac{(c_{v0}'+B')\eta_0'}{\lambda_0'}$$

$$M=\frac{U'}{\sqrt{\gamma_0B'T_c'}},\quad \Theta=\frac{\Theta'}{T'_c},\quad \gamma_0=1+\frac{B'}{c_{v0}'}$$

Primes are used for dimensional values, subscript «c» for critical values, subscript «0» for a perfect gas. Parameter of hydrostatic compressibility ε_g is related with parameters defined above as $\varepsilon_g=\gamma_0M^2Ra/(\mathrm{Pr}\,\Theta\,\mathrm{Re}^2)$.

The model with two-scale splitting of the pressure presented initially for a perfect gas [A.G. Churbanov, A.N. Pavlov, A.V. Voronkov, and A.A. Ionkin. Prediction of low Mach number flows: a comparison of pressure-based algorithms. *Proceeding of the Tenth International Conference on Numerical Methods in Laminar and Turbulent Flow, Swansea, 1997* (Pineridge Press, Swansea, 1997), p. 1099] was developed by E. Soboleva and her colleagues for non-perfect media, particularly, for near-critical fluids. On the basis of this model both low-Mach-number flows and acoustic waves can be simulated effectively.

Another model based on the approached Navier-Stokes equations in approximation to low-speed flows with acoustic “filtering” was used earlier. This model presented firstly in [Лапин Ю.В., Стрелец М.Х. Внутренние течения газовых смесей. М.: Наука, 1989. 368 с.] and [Paolucci S. On the filtering of sound from the Navier-Stokes equations. *Sandia Nat. Lab. Rep*. SAND 82-8257. 1982. 52 p.] was applied to simulations of large-scale motions of near-critical fluids with low Mach number.

In examples below a near-critical fluid is described by the Van der Waals equation of state $P = \rho T/(1-b\rho) - a\rho^2$ ($a = 9/8$ and $b = 1/3$). When $a = 0$ and $b = 0$, this equation is transformed into the equation of state $P = \rho T$ for a perfect gas. Now the extended models including the different equations of state are developed as well.

See [6, 16, 17] in PUBLICATIONS

Back