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Numerical modeling of a low-velocity microparticle's spraying process in a heated gas stream

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Abstract. The paper studies a building-up process where low-velocity heated micro-particles collide with an undeformed substrate. For a model of a one-dimensional rigid plastic collision with the law of hardening close to linear law approximate analytical estimates for the final thickness of a deformed hot particle incident on an undeformed substrate, its radius and the collision time as a function of the velocity are obtained. A comparison of the numerical and analytical results in the one-dimensional approximation with the results of multidimensional modeling of the unsteady collision process using finite-element schemes is carried out. The possibility of usage the one-dimensional model is shown to estimate layers' thickness as a function of micro-particles' velocity and gas' temperature. Also, the solution for a model of a one-dimensional rigidviscoplastic collision with a non-linear law of viscosity is obtained.

1. Introduction

The spraying process of micro-particles is commonly used to reinforce responsible structural elements or to build-up layer by layer an element that was damaged during a process of exploitation. For this purpose some amount of micro-particles is distributed within a heated gas stream moving towards a mobile substrate. Yield stress of a particle's material is rather low due to high temperature of a stream which is just below the material's melting point. As the result, a laminate texture of the structure element is formed. The thickness of the layers and mechanical properties of the built-up by spraying element depend on ultimate particles' thickness under the dynamic deformation process.

A formulation of the problem and method of calculation the ultimate particle's thickness are like the Taylor model for one-dimensional collision of a bar from rigid plastic material with an undeformed substrate [1]. However, the applicability of such a simplified model of impact interaction is not justified in advance.

The system of ordinary differential equations that are corresponding to the considered approximate model was solved numerically by a finite difference calculation. The applicability of approach of using a one-dimensional model to estimate the thickness of built-up layers as a function of gas' temperature and particles' velocity was approved by comparison with a multidimensional elastoplastic FEM calculation.

Also, the solution for a model of a one-dimensional rigidviscoplastic collision for the non-linear law of viscosity was obtained. It is shown that the same particle's dimensions, as if using the previous model, occur under the higher temperature due to dynamic rise of the yield stress.

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2. The Taylor model

Let us consider a dynamic deformation pattern that corresponds to the classical Taylor model and was given in [1-3]. A bar of length h_0, radius r_0, made of rigid plastic material is moving at a velocity of v_0 towards an undeformed substrate. At $t = 0$ the bar strikes the substrate (figure 1). We consider a movement of the bar as a quasi-one-dimensional; velocity and strain distributions are homogenous within every cross-section of the bar.

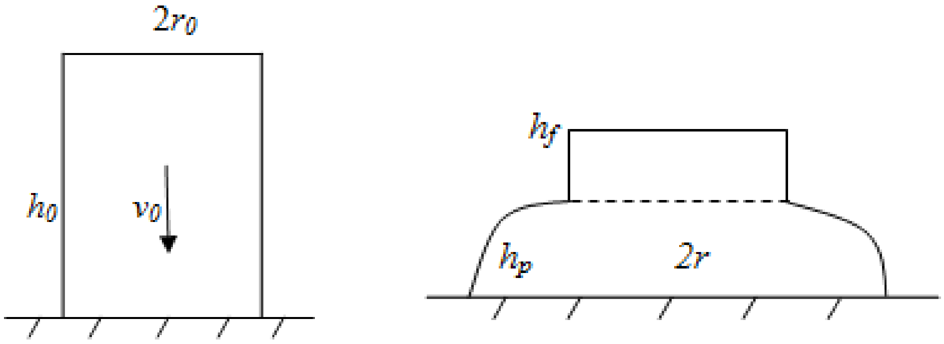


Figure 1. The Taylor collision model.

A system of equations comprises a relation for deformation behind a plastic wave, the conservation of momentum at the plastic wave front, a kinematic relation for a height of bar's rigid part, the second Newton's law for the rigid part, and a plastic deformation law:

$$\varepsilon = v/(v+w)$$

$$\rho(v+w)v = \sigma - \sigma_{\mathrm{s}}$$

$$\mathrm{d}h/\mathrm{d}t = -(v+w)$$

$$\rho h \frac{\mathrm{d}v}{\mathrm{d}t} = -\sigma_{\mathrm{s}}$$

$$\sigma = \sigma_{\mathrm{s}} + E_{\mathrm{p}}\varepsilon^m$$

where $v(t)$ is rigid part's velocity; $w(t)$ is plastic wave velocity; $\sigma(t)$ is stress behind the shock wave; σ_{s} is a yield stress; ρ is a density; E_{p} is a strain hardening modulus; m is a degree coefficient of hardening. Initial conditions are:

$$t = 0:\ v = v_0,\ h = h_0$$

Conditions to determine a stopping time $t = t_f$ and the rigid part's height h_f are:

$$v = 0,\ w = 0$$

One can transform the considered system of equations to:

$$\mathrm{d}h/\mathrm{d}t = -(v+w)$$
$$\mathrm{d}v/\mathrm{d}t = -\sigma_s/(\rho h)$$
$$w = \left(E_p/\rho\right)^{1/(m+1)} v^{(m-1)/(m+1)} - v$$

It is impossible to solve analytically this system of equations for the arbitrary m. Therefore an approximate solution was made with the principal terms obtained at the point with $m = 1+\gamma > 1$ where $\gamma << 1$. It was assumed that the $\sigma - \varepsilon$ hardening law within a plastic domain is close to linear ($E_p << E$ where E is a Young's modulus).

The approximate solution is:

$$h(v) = h_0 \exp((v - v_0)/v_*)$$
$$v(t) = v_0 + v_* \ln(1 - t/t_0)$$
$$w(t) = v_c((v/v_c)^{\gamma/2} - v/v_c)$$
$$h_f = h_0 \exp(-v_0/v_*)$$
$$t_f = t_0(1 - \exp(-v_0/v_*))$$

where $v_c = \left(\dfrac{E_p}{\rho}\right)^{1/2}$, $v_* = \dfrac{\sigma_s}{\left(\rho E_p\right)^{1/2}}$, $t_0 = h_0/v_c$.

One can obtain the plastic zone's height h_p using the plastic wave velocity $w(t)$:

$$h_p = \int_0^{t_f} w(t)\mathrm{d}t = h_0\left(\left(\frac{v_0}{v_c}\right)^{\gamma/2}\left[1 - \exp\left(\frac{-v_0}{v_*}\right)\right] - \frac{v_0}{v_c} + \frac{v_*}{v_c}\left[1 - \exp\left(\frac{-v_0}{v_*}\right)\right]\right)$$

Total bar's height is $h_1 = h_p + h_f$. It is important to notice that shape of bar's part in plastic state is not defined according to the current approach. Therefore the shape is defined by using the assumption of incompressibility of bar's material: $r = r_0\sqrt{(h_0 - h_f)/h_p}$.

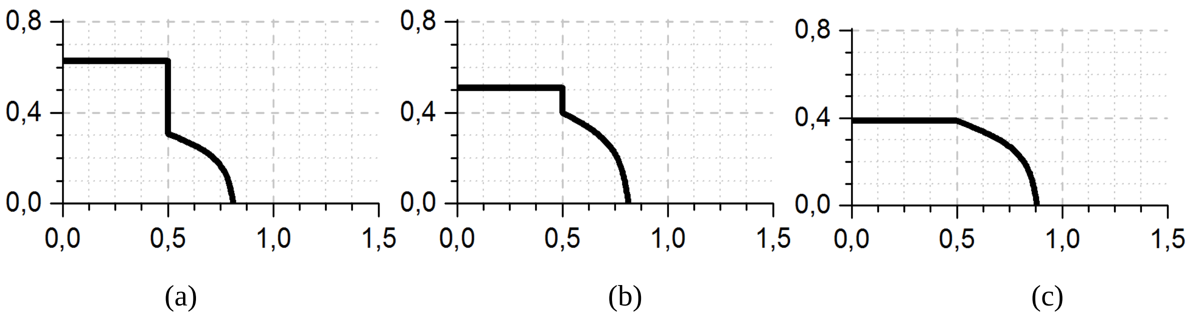


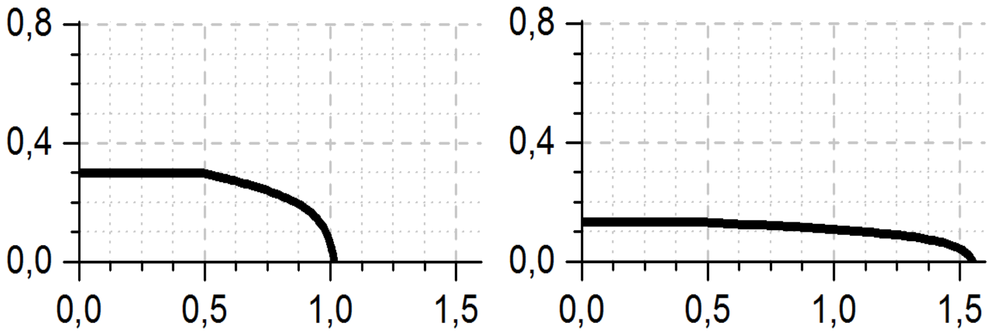
Figure 2. Particle's shape at v_0 = 20 m/s, $E_p = 2.5\cdot10^7$ Pa, m = 1.2;
(a) T = 1600; (b) T = 1620; (c) T = 1633.

Using the primary formulas for the numerical evaluation of the bar's shape a number of initial conditions have been analyzed. Listed below parameters were used: particle's material is titanium;

height $h_0 = 30 \cdot 10^{-6}\, m$; $r_0 = h_0 / 2$; density $\rho =$ 4500 kg/m^3; Young's modulus $E = 116 \cdot 10^9$ Pa; Poisson ratio ν = 0.32; yield stress at normal conditions $\sigma_{s0} = 3 \cdot 10^8$ Pa; initial velocity v_0 = 20 m/s. The bar's temperature is within the range T = 1600 - 1660 °C, whereas titanium's melting point T_m is 1665 °C. Defining yield stress at the temperature about the melting point is a rather uncertain task [4]; thus a linear approximation $\sigma_s = \max\left(\sigma_{s0}\left(T_m - T\right)/T_m, 0\right)$ was used.

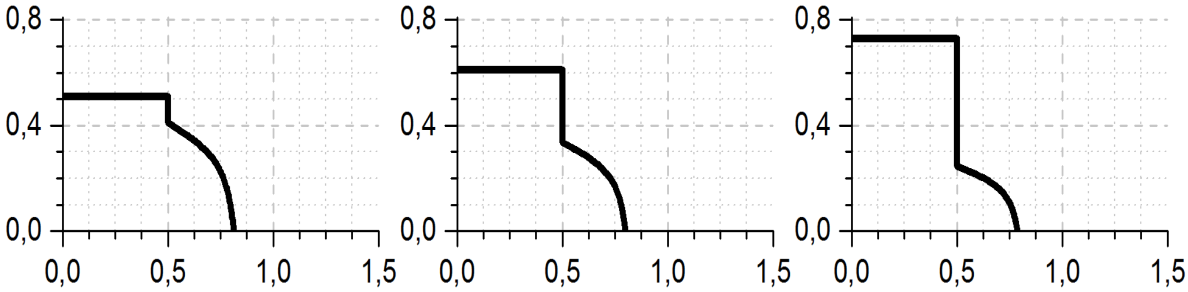
Figures 2 and 3 show the shape after collisions at different stream's temperatures, which are close to the melting point.

Figure 4 shows the shape after collisions at different stream's velocities. Figure 5 shows an influence of the strain hardening modulus and the power of hardening on the shape.



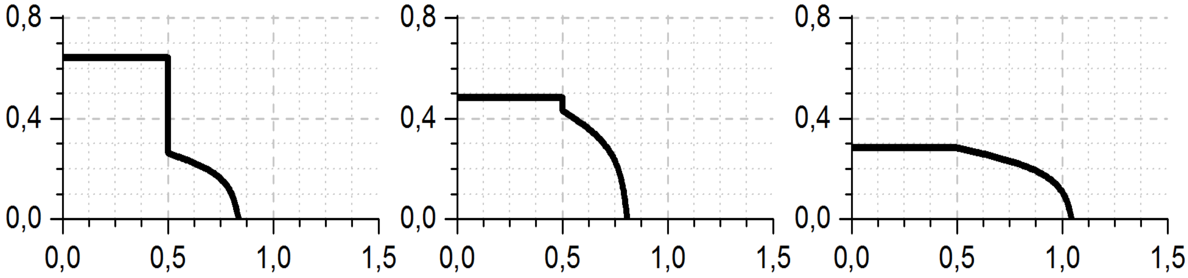
(a) (b)

Figure 3. Particle's shape at v_0 = 20 m/s, $E_p = 2.5 \cdot 10^7$ Pa, m = 1.2;
(a) T = 1640; (b) T = 1650.



(a) (b) (c)

Figure 4. Particle's shape at T = 1620, $E_p = 2.5 \cdot 10^7$ Pa, m = 1.2;
(a) v_0 = 20 m/s; (b) v_0 = 15 m/s; (c) v_0 = 10 m/s.

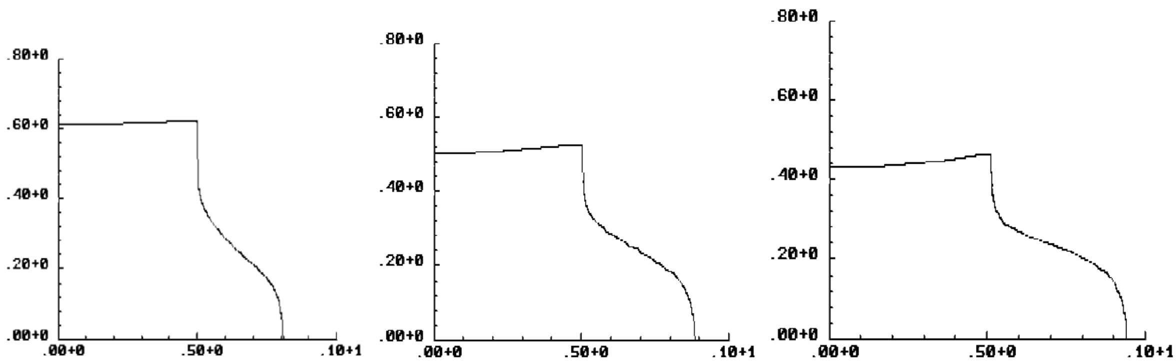


(a) (b) (c)

Figure 5. Particle's shape at v_0 = 20 m/s, T = 1620;
(a) $E_p = 2.5 \cdot 10^7$ Pa, m = 1.8; (b) $E_p = 1 \cdot 10^7$ Pa, m = 1.8; (c) $E_p = 1 \cdot 10^7$ Pa, m = 1.2.

3. Multidimensional modeling

In order to compare the above results a multidimensional modeling based on the numerical method from [5-7] was performed. Techniques to improve the accuracy of numerical solutions with complex time-varying geometry and reduce their computational costs were applied. The approach combines shock-capturing computations with the following methods: overlapping meshes for specifying complex geometry; elastic arbitrarily moving adaptive meshes for minimizing the approximation errors near shock waves, boundary layers, contact discontinuities, and moving boundaries; matrix-free implementation of efficient iterative and explicit-implicit finite element schemes; balancing viscosity (a version of the stabilized Petrov–Galerkin method); exponential adjustment of physical viscosity coefficients; and stepwise correction of solutions for providing their monotonicity and conservativeness. The grid-characteristic method was used in [8-9] to solve contact problems as well. Figures 6 and 7 show ultimate bar's shapes for the same as figure 2 initial conditions.



(a) (b) (c)

Figure 6. Particle's shape at v_0 = 20 m/s;
(a) T = 1600; (b) T = 1620; (c) T = 1633.

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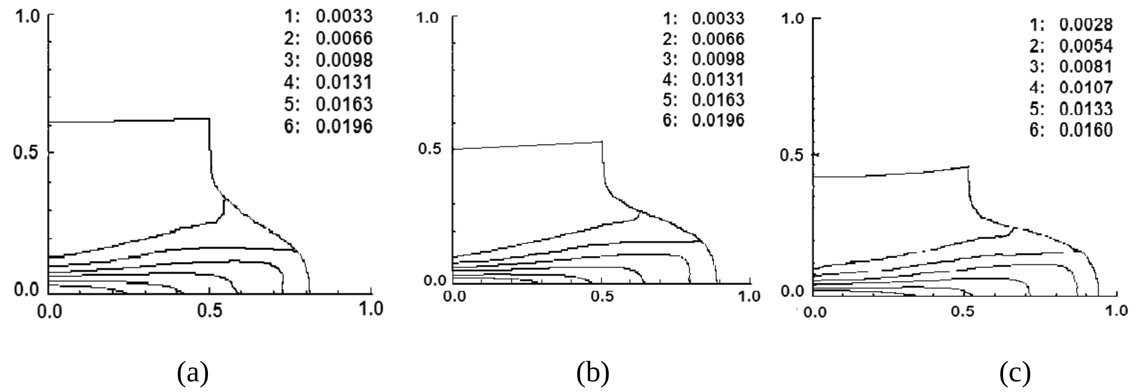


Figure 7. Contour plot of the plastic work at v_0 = 20 m/s;
(a) T = 1600; (b) T = 1620; (c) T = 1633.

Comparing figure 2 and figure 6 one can see the one-dimensional model gets ultimate thickness on the bar's part in a plastic state rather well within studied boundaries comparing to the multidimensional one.

4. The Ishlinsky model

In [8] Ishlinsky proposed the quasi-one-dimensional model based on a rigidviscoplastic material with a non-linear law of viscosity, so called rigid viscoplastic hardening law. Let a bar of length l made of such material is moving at a velocity of v_0 towards an undeformed substrate. At $t = 0$ the bar strikes the substrate (figure 8). We consider a motion of the bar as a quasi-one-dimensional. Velocity and strain distributions are homogenous within every cross-section of the bar. Let's put down a system of equation of the model.

For the viscoplastic part of a bar ($|\sigma| > \sigma_s$) in any cross section the velocity $v(x,t)$ is:

$$\rho \frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x}$$

Strain rate depends on stress by:

$$\sigma = -\sigma_s + \mu \frac{\partial v}{\partial x} \text{ while } |\sigma| > \sigma_s$$

$$\frac{\partial v}{\partial x} = 0 \text{ while } |\sigma| \le \sigma_s$$

Excluding stress one can obtain the equation for velocity:

$$\frac{\partial v}{\partial t} = \frac{\mu}{\rho} \frac{\partial^2 v}{\partial x^2} \text{ while } 0 \le x < x_0(t)$$

For motion of the rigid bar's part with decreasing velocity $-v_0(t)$ an equation is:

$$\frac{\mathrm{d}v_0}{\mathrm{d}t} = \frac{-\sigma_s}{\rho\left(l - x_0(t)\right)}$$

Owing to continuity of velocity and stress at the moving boundary $x = x_0(t)$ between viscoplastic and rigid parts we have

$$v\left[x_0(t), t\right] = -v_0(t)$$

$$\frac{\partial}{\partial x} v\left[x_0(t), t\right] = 0$$

Boundary conditions for $v(x,t)$ are:

$$t > 0 \; v(0,t) = 0$$

Initial conditions are:

$$0 < x \le l \; v(x,0) = -V_0$$

For the rigid part the initial condition is:

$$v_0(0) = V_0$$

Position of the moving boundary at initial time moment is:

$$x_0(0) = 0$$

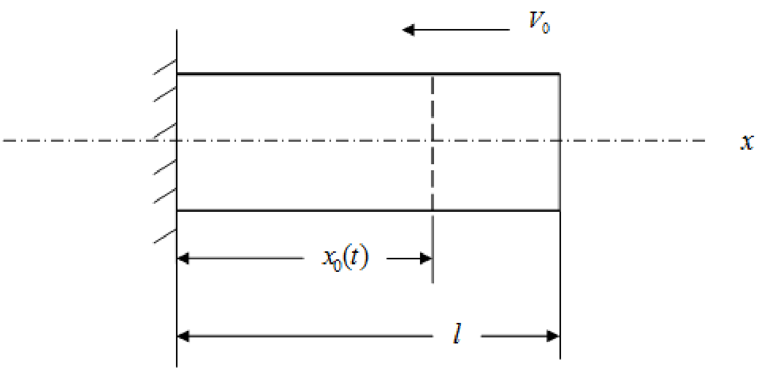


Figure 8. System's state at the initial time moment.

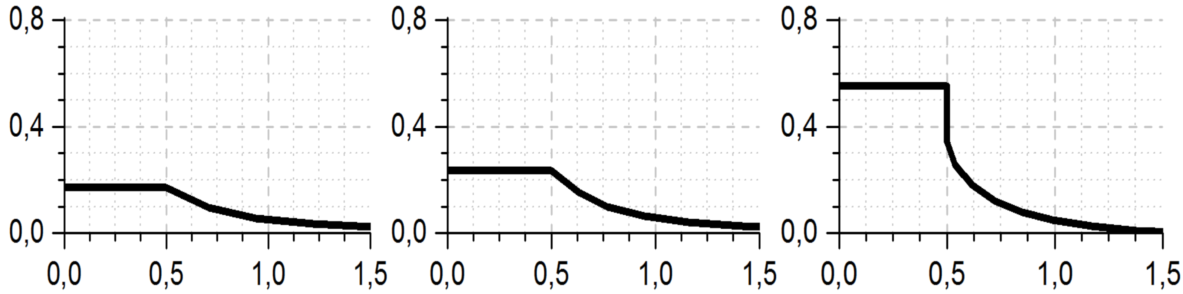
Here we have a non-classic heat conduction problem with an undefined internal moving boundary layer for the velocity of a rigid part $v_0(t)$ and its position $x_0(t)$ and velocity $v(x,t)$ of any cross section lying between a substrate and the internal moving boundary layer. We have found that the Saint-Venant's parameter $s = \sigma_s l / \mu V_0$ has a crucial influence on the system behavior.
Due to incompressibility of the material one can calculate an area F of any cross section:

$$F = F_0\left(1 + \frac{\partial U}{\partial x}\right)^{-1}$$

where U is the longitudinal shift of a cross section from its original position within a viscoplastic part:

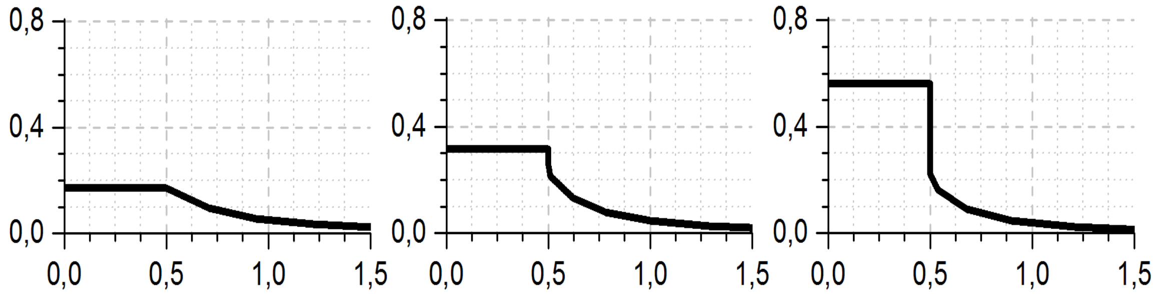
$$U = \int_0^t v(x,\tau)\mathrm{d}\tau$$

It is of interest to examine a bar with the same geometrical and material parameters as previously, i.e. particle's material is titanium; height l = 1 (unitless); $r_0 = l/2$; density ρ = 4500 kg/m3; yield stress at normal conditions $\sigma_{s0} = 3\cdot 10^8$ Pa;
The problem was solved numerically. The dependence of the shape of a specimen as a function of its velocity, temperature and viscosity are presented (figures 9-11).



(a) (b) (c)

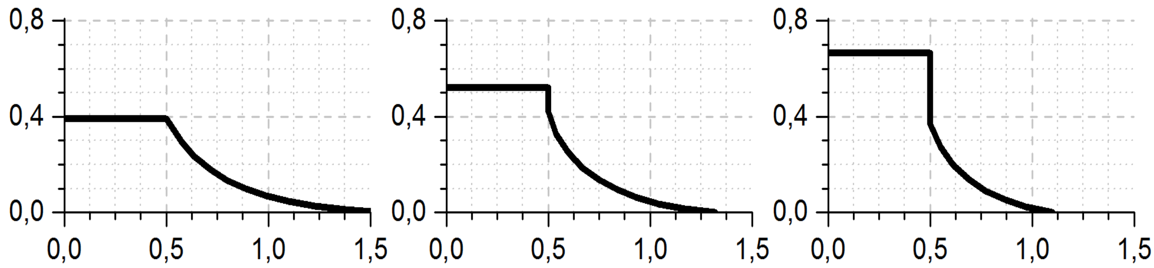
Figure 9. Particle's shape at T = 1664, μ = 0.3;
(a) V_0 = 20m/s; (b) V_0 = 15 m/s, (c) V_0 = 10 m/s.



(a) (b) (c)

Figure 10. Particle's shape at V_0 = 20m/s, μ = 0.3;
(a) T = 1664; (b) T = 1662; (c) T = 1660.

It is seen that high influence of stream temperature and viscosity on a particle's ultimate shape takes place. Varying the parameters one is able to change the shape drastically. Comparing the Ishlinsky and the Taylor models shows that an influence of viscoplasticity at the same initial conditions reveals by the less value of plastic deformations in the Ishlinsky model than in the Taylor one. The reason is the dynamic rise of the yield stress.



(a) (b) (c)

Figure 11. Particle's shape at V_0 = 20m/s, μ = 0.9;
(a) T = 1664; (b) T = 1662; (c) T = 1660.

5. Conclusions

The spraying process of micro-particles is commonly used to reinforce responsible structural elements or to build-up layer by layer an element that was damaged during a process of exploitation. The possibility of usage the one-dimensional model is shown to estimate layers' thickness as a function of micro-particles' velocity and gas' temperature.

For the laws of hardening close to linear by the perturbation method the approximate analytical estimates are obtained for the finite thickness of a deformed hot particle incident on an undeformed substrate. Also, the estimates are obtained of the ultimate particle's radius and the collision time as a function of the velocity. The system of ordinary differential equations of the rigid plastic model under consideration is numerically and analytically solved.

A comparison of the numerical and analytical results in the one-dimensional approximation with the results of multidimensional elastoplastic modeling of the unsteady collision process using finite-element schemes is carried out. Techniques to improve the accuracy of numerical solutions with complex time-varying geometry and reduce their computational costs were applied.

The efficiency of the obtained approximate formulas for estimating the thickness of the build-up layers as a function of particle velocity and flow temperature is shown. Also, the solution for a model of a one-dimensional rigidviscoplastic collision with the non-linear law of viscosity was obtained. It is shown that the same particle's dimensions, as in the case of the previous model, are observed under the higher temperature while taking into account a non-linear law of viscosity due to dynamic rise of the yield stress.

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