Mathematical modelling of the VHCF damage development in smooth specimens under arbitrary loading mode

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INTRODUCTION

The fatigue design of the most structural elements in based on results of low- and high cycle fatigue that are limited by 5×10^4 and 10^7 cycles, respectively. But for example in the case of aircraft industry, the typical loading frequency of blades in gas-turbine engine compressor can reach 4500 Hz [1]. Corresponding number of cycles can reach up to 10^{10}. This estimated durability is far beyond the low- and high cycle fatigue range, which is in the range of very-high cycle fatigue. Since it is assumed that a VHCF criterion should be a physically based one and contains some structural parameters. Nonetheless, it is important to develop an engineering criterion. The main idea for such type of criterion is generalization of popular multiaxial criterion to the range of VHCF. Some works [2] in this direction have shown an appropriate result. Such approach needs some axial experimental data under different stress ratios to determine the parameters of the model. The model should be verified by prediction the result of VHCF tests under complex, multiaxial loading. Therefore, developing and modeling of new multiaxial loading schemes is an important subject in the framework of an engineering criterion development.

MULTI-REGIME MODEL OF FATIGUE DAMAGE ACCUMULATION

Investigation on fracture surfaces of specimens or structures subjected to cyclic loading usually shows a complex scenario of fatigue crack development i.e., crack opening mode is changing at different stages of crack growth under multiaxial loading or various loading schemes (tension-compression, tension- tension, torsion, three-point bending) in the VHCF regime.

Taking into account these features of fatigue crack initiation and growth the multi-regime model is proposed. The model combines the modern approach of SN-curve, special algorithm of SN-curve branch determination, kinetic equation for damage function evolution and multi-axial criterion as a closing equation.

The following scheme of the amplitude fatigue curve is used. Up to value $N \sim 10^3$ the regime of repeated-static loading with amplitude slightly differing from the static tensile strength σ_B is realized. Further the fatigue curve (Wöhler curve or left branch of fatigue curve) describes the modes of the LCF-HCF up to $N \sim 10^7$ with an asymptotic exit to the fatigue limit σ_u Then, a zone of change in fracture mechanisms begins and a further drop in fatigue strength, starting from values of $N \sim 10^8$, to a new limiting value σ_u% in accordance with the right branch of the bimodal fatigue *S-N* curve (VHCF mode) [3].

Kinetic equation for damage in LCF-HCF mode

Various criteria use different stress combinations to calculate an equivalent stress value. Some of them based on normal stress components of a stress state while other based on shear components. In this paper we are going to implement two criteria simultaneously: one is based on a normal opening micro-cracks which is the stress-based Smith–Watson–Topper (SWT) [4,5], the other one is based on a shear micro-cracks and implements the notion of a critical plane which is the stress-based Carpinteri–Spagnoli–Vantadori (CSV) [6].

The fatigue fracture criterion corresponding to the left branch of the bimodal fatigue curve in the following has the form:

$$\sigma_{eq} = \sigma_u + \sigma_L N^{-\beta_L}$$

From the condition of repeated-static fracture up to values of $N \sim 10^3$ by the method [2] it is possible to obtain the value $\sigma_L = 10^{3\beta}(\sigma_B - \sigma_u)$. In these formulas σ_B is the static tensile strength of the material, σ_u is the classic fatigue limit of the material during a reverse cycle (asymmetry coefficient of the cycle R = -1), β_L is power index of the left branch of the bimodal fatigue curve.

In order to describe the process of fatigue damage development in the LCF-HCF mode, a damage function $0 \le \psi(N) \le 1$ is introduced, which describes the process of gradual cyclic material failure. When $\psi = 1$ a material particle is considered completely destroyed. Its Lamé elastic moduli become equal to zero. The damage function ψ as a function on the number of loading cycles for the LCF-HCF mode is described by the kinetic equation [3]:

$$\partial\psi / \partial N = B_L \psi^{\gamma} / (1 - \psi^{\alpha})$$

where α and $0 < \gamma < 1$ are the model parameters that determine the rate of fatigue damage development.

An equation for damage of a similar type was considered in [7], its numerous parameters and coefficients were determined indirectly from the results of uniaxial fatigue tests. In our case, the coefficient B_L is determined by the procedure that is clearly associated with the selected criterion for multiaxial fatigue failure of one type or another. It has the following form.

The expression for the coefficient B_L has a form [3]:

$$B_L = 10^{-3}\left[\left\langle \sigma_{eq} - \sigma_u \right\rangle / (\sigma_B - \sigma_u)\right]^{1/\beta_L} \alpha / (1 + \alpha - \gamma) / (1 - \gamma)$$

where the value σ_{eq} is determined by the selected mechanism of fatigue failure and the corresponding multiaxial criterion.

There are not one but two B_L values, namely B_L^n and B_L^τ. They have the forms:

$$B_L^n = 10^{-3}\left[\left\langle \sigma^n - \sigma_u \right\rangle / (\sigma_B - \sigma_u)\right]^{1/\beta_L} \alpha / (1+\alpha-\gamma)/(1-\gamma)$$

$$B_L^\tau = 10^{-3}\left[\left\langle \sigma^\tau - \sigma_u \right\rangle / (\sigma_B - \sigma_u)\right]^{1/\beta_L} \alpha / (1+\alpha-\gamma)/(1-\gamma)$$

It means there are 2 damage values $\psi^n = f(B_L^n)$ and $\psi^\tau = f(B_L^\tau)$. We will assume that the choice is determined by the mechanisms of microcracks development and fatigue fracture criteria SWT and CSV. For microcracks of normal opening $\sigma_{eq} = \sigma^n$, for shear $\sigma_{eq} = \sigma^\tau$. The resulting formulas for the coefficients of the kinetic equation for damage operate in the range $\sigma_u < \sigma_{eq} \le \sigma_B$.

Kinetic equation for damage in VHCF mode

The criterion for multiaxial fatigue failure in the VHCF mode corresponding to the right branch of the bimodal fatigue curve has the form:

$$\sigma_{eq} = \tilde{\sigma}_u + \sigma_V N^{-\beta_V}$$

We will assume that the choice σ_{eq} is determined by the same mechanisms of microcracks development and fatigue fracture criteria SWT and CSV as in the HCF mode, $\sigma_{eq} = \sigma^n$ or $\sigma_{eq} = \sigma^\tau$.

From the condition of similarity of the reference points for the left and right branches of the bimodal fatigue curve [2], one can obtain the formula $\sigma_V = 10^{8\beta_V}(\sigma_u - \tilde{\sigma}_u)$.

Here $\tilde{\sigma}_u$ is the fatigue limit of the material in the reverse cycle for the VHCF mode, β_V is the power exponent of the right branch of the bimodal fatigue curve.

For the VHCF mode the evolutionary equation for damage has the same form:

$$d\psi / dN = B_V \psi^\gamma / (1-\psi^\alpha),\ 0 < \alpha,\ \gamma < 1$$

As in the previous section, we can obtain expressions for the coefficients of the kinetic equation of damage in the VHCF mode [8]:

$$B_V^n = 10^{-8}\left[\left\langle \sigma^n - \tilde{\sigma}_u \right\rangle / (\sigma_u - \tilde{\sigma}_u)\right]^{1/\beta_V} \alpha / (1+\alpha-\gamma)/(1-\gamma)$$

$$B_V^\tau = 10^{-8}\left[\left\langle \sigma^\tau - \tilde{\sigma}_u \right\rangle / (\sigma_u - \tilde{\sigma}_u)\right]^{1/\beta_V} \alpha / (1+\alpha-\gamma)/(1-\gamma)$$

These resulting formulas for the coefficients operate in the range $\tilde{\sigma}_u < \sigma_{eq} \le \sigma_u$.

Multiaxial criterion for multi-regime model

The presented model can be used with different multiaxial criteria depending on material's fatigue behavior. We propose to use the multiaxial criterion as 'close equation' for damage function.

The criterion SWT of multiaxial fatigue failure in the LCF-HCF mode with the development of normal-stress micro-cracks (stress-based SWT) corresponding to the left branch of the bimodal fatigue curve has the form:

$$\sqrt{\langle \sigma_{1_{\max}} \rangle \Delta\sigma_1 / 2} = \sigma_u + \sigma_L N^{-\beta_L}$$

where σ_1 is the largest principal stress, $\Delta\sigma_1$ is the spread of the largest principal stress per cycle, $\Delta\sigma_1 / 2$ is its amplitude. According to the chosen criterion only tensile stresses lead to failure, so it has the value $\langle \sigma_{1_{\max}} \rangle = \sigma_{1_{\max}} H(\sigma_{1_{\max}})$. Thus for chosen criterion $\sigma^n = \sqrt{\langle \sigma_{1_{\max}} \rangle \Delta\sigma_1 / 2}$

The criterion CSV of multiaxial fatigue failure in the LCF-HCF mode, including the concept of a critical plane, in its simplest variant has the form:

$$\sqrt{(\langle \Delta\sigma_n \rangle / 2)^2 + 3(\Delta\tau_n / 2)^2} = \sigma_u + \sigma_L N^{-\beta_L}$$

where $\Delta\tau_n / 2$ is the amplitude of the tangential stress on the plane (critical), where it reaches its maximum value, $\Delta\sigma_n / 2$ is the amplitude of the normal (tensile) stress on the critical plane, $\langle \Delta\sigma_n \rangle = \Delta\sigma_n H(\sigma_{n_{\max}})$. This criterion includes the mechanism of fatigue fracture with the formation of shear micro-cracks. For chosen criterion $\sigma^\tau = \sqrt{(\langle \Delta\sigma_n \rangle / 2)^2 + 3(\Delta\tau_n / 2)^2}$

Material properties change

The nonlinear dependence of the Lamé elastic moduli on the damage function is chosen in the following form:

$$\lambda(\psi) = \lambda_0 (1 - \psi^\kappa), \;\; \mu(\psi) = \mu_0 (1 - \psi^\kappa)$$

When $\psi = 1$ a material particle is considered completely destroyed. Its Lamé elastic moduli become equal to zero. In the numerical procedure nodes with damage $\psi = 1$ are removed from the calculation area and form a localized zone (crack-like zone) of completely destroyed material.

CALCULATION ALGORITHM

A uniform numerical method has been developed based on the integration of the differential equation for damage [3,8]. The damage function approximation was applied at the *k*-node of the computational grid for given discrete values ψ_k^t at moments N^t and sought ψ_k^{t+1} at moments

N^{t+1}. To analytically integrate the damage equation, the value $\alpha = 1-\gamma$ was chosen. The explicit expression for $\psi_k^{t+1}(\psi_k^t, \Delta N^t)$ has the form [3]:

$$\psi_k^{t+1} = \left(1 - \sqrt{\left(1-(\psi_k^t)^{1-\gamma}\right)^2 - 2(1-\gamma)B_k \Delta N^t}\right)^{1/(1-\gamma)}$$

Here increment value ΔN^t defined as follows. Based on the stress state calculation data for the current number of cycles, the coefficient B_k is calculated for each node. After that, for each node, the following values are calculated

$$\Delta N_k^t = 0.25(1-(\psi_k^t)^{1-\gamma})^2 \,/\, 2 \,/\, (1-\gamma) \,/\, B_k$$

The increment of the number of loading cycles is $\Delta N^t = \min_k \Delta N_k^t$.

For each node, based on its current level of damage and equivalent stress, a new level of damage is found taking into account the calculated increment ΔN^t. Then the new Lamé moduli are calculated:

RESULTS OF NUMERICAL SIMULATION

For verification of the proposed model the torsion VHCF tests were used. The VHCF tests were performed by using a direct piezoelectric fatigue torsion machine [9]. The loading frequency is about 20 kHz. The specimen geometry is hourglass with smooth gage section. The material for torsion tests is titanium alloy with following characteristics: Young's modulus is 115 GPa, density 4500 kg/m^3, Poisson ratio is 0.3, yield stress 980 MPa, fatigue limit 440 MPa, VHCF fatigue limit is 385 MPa. The results show a permanent decreasing of cyclic strength with number of cycles. Also the results of experimental investigation have shown [10] that crack initiation and early growth is being in the plane of maximum shear stress orientated by 0 or 90 degree by the specimen axis. Therefore, there is a transition from crack initiation by the shear mechanisms to crack propagation by the normal crack opening mode.

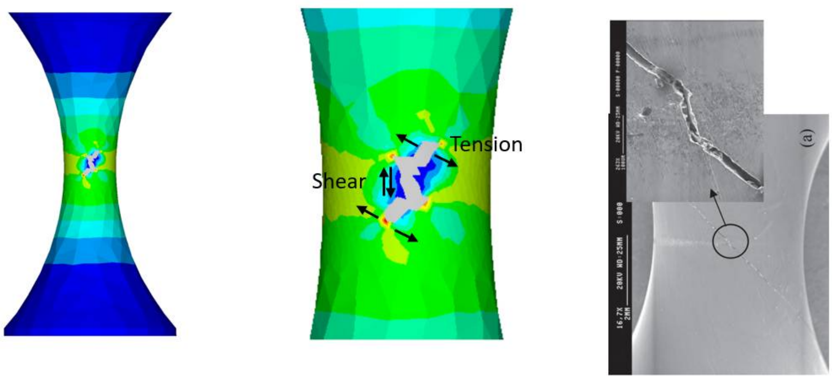

Fig. 1. The crack path simulation under pure VHCF torsion loading (a)-(b) and result of VHCF test (c)

The results of damage zone development in hourglass specimen under pure VHCF loading are presented on figure 1. The macroscopic view shows a typical for torsion loading a zigzag crack path, figure 1-a. The details of this crack are presented on figure 1-b. The special algorithm allows to count the number of elements degraded by a given mechanisms. It was found that at the initial stage of damage development the elements were mostly degraded due to shear mechanisms. This is corresponding to vertical crack along the specimen axis. Later, the dominant mechanism for mechanism degradation is normal crack opening. This period is corresponding to inclined part of crack path in the plane of maximum normal stress. Comparison of these results of numerical simulations with experimental results, figure 1-c shows a qualitative coincidence of crack paths. It is good result since there are not any numerical switches inside the algorithm. Therefore, the model shows nice agreement with experimental results under multiaxial loading and allows estimating the number of cycles for crack initiation and duration of crack growth, predicting the shape of fatigue damage zone at different stages.

CONCLUSIONS

The paper introduces the multi-mode model of fatigue damage accumulation based on damage theory and multiaxial criteria. The proposed approach does not need to introduce an initial crack before the numerical simulation. The model can be combines with different multiaxial criterion with different driving mechanisms of failure such as normal crack opening, shear cracks. The model can be used to describe the degradation of the material under arbitrary stress state and can be applied to predict the crack initiation under shear condition and sporadic transition to the normal opening crack growth under VHCF pure torsion loading in the smooth hourglass specimens. The results of calculation show good agreement with experimental results. Therefore, the proposed approach is powerful instrument to investigate the structural integrity of complex engineering elements.

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REFERENCES

[1] Shanyavskiy A., *Science China Physics, Mechanics and Astronomy.* 2014; 57(1): 19–29
[2] Burago N. G. and Nikitin I. S., *Mathematical Modeling and Optimization of Complex Structures,* Heidelberg: Springer. 2016; 117–130
[3] Nikitin I. S., Burago N. G., Zhuravlev A. B. and Nikitin A. D., *Mechanics of Solids.* 2020; 55(8): 298–306
[4] Smith R. N., Watson P. and Topper T.H., *J. of Materials.* 1970; 5: 767–778
[5] Gates N. and Fatemi A., *Int. J. of fatigue.* 2016; 91: 337–351
[6] Carpinteri A., Spagnoli A. and Vantadori S., *Int. J. of Fatigue.* 2011; 33: 969–976
[7] Marmi A. K., Habraken A. M. and Duchene L., *Int. J. of fatigue.* 2009; 31: 2031–2040
[8] Nikitin I. S., Burago N. G., Nikitin A. D. and Stratula B. A., *AIP Conference Proceedings.* 2020; 2321: 050015
[9] Nikitin A., Bathias C. and Palin-Luc T., *Fatigue and Fracture of Engineering Materials and Structures.* 2015; 38(11): 1294–1304
[10] Nikitin A., Palin-Luc T., Shanyavskiy A. and Bathias C., *Engineering Fracture Mechanics.* 2016; 167: 1125–1132