

Modelling of the microparticle spraying process in a heated gas stream

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Abstract. Approximate analytical estimates for the final thickness of a deformed hot particle incident on an undeformed substrate, its radius and the collision time as a function of the speed are obtained for the law of hardening close to linear by the perturbation method. The system of ordinary differential equations of the approximate model under consideration is numerically and analytically solved. A comparison of the numerical and analytical results in the one-dimensional approximation with the results of multidimensional modelling of the unsteady collision process using finite-element schemes is carried out.

1. Introduction

The result of a building-up process such as layers thickness and mechanical properties are determined by thickness of almost melted particles after collisions. The Taylor model for single-dimensional collision of a pure plastic bar with an undeformed substrate was applied to model the process. In case of a hardening process similar to a linear one the approximate analytical evaluation of particle's final thickness, radius, and impact time as function of particle's speed before the collision were conducted using the perturbation method. This approach was approved by comparison with a multidimensional FEM calculation.

2. The Taylor model

Let us consider a dynamic deformation pattern that corresponds to the classical Taylor's model and was given in [1-3]. A bar of length h_0 , radius r_0 , and made of pure plastic material is moving at a velocity of v_0 (the velocity vector and bar's side with given length are parallel) towards an undeformed substrate. At $t=0$ the bar hits the substrate that is normal to the velocity vector (Fig. 1). We consider a movement of the bar as a quasi-single-dimensional; speed and strain distributions are homogenous within an every cross-section of the bar.



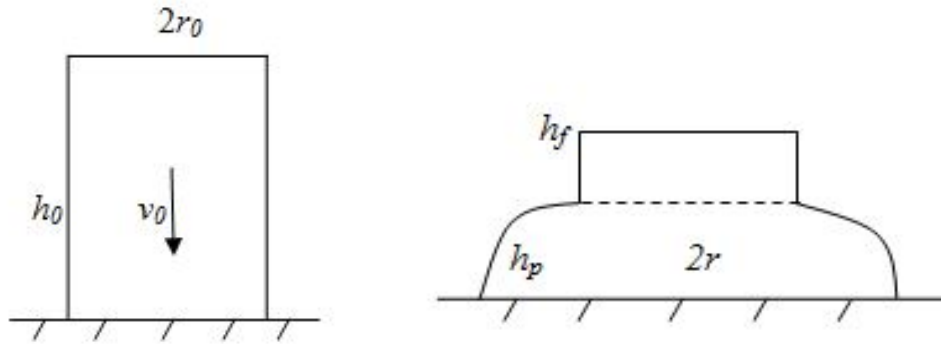


Figure 1. The Taylor collision model

A system of equations involves a relation for deformation behind a plastic wave, the conservation of momentum at the plastic wave front, a kinematic relation for a height of bar's intact part, the Newton's law for the intact part, and a plastic deformation law:

$$\varepsilon = v / (v + w)$$

$$\rho(v + w)v = \sigma - \sigma_s$$

$$dh / dt = -(v + w)$$

$$\rho h dv / dt = -\sigma_s$$

$$\sigma = \sigma_s + E_p \varepsilon^m$$

where $v(t)$ is intact part's velocity; $w(t)$ is plastic wave speed; $\sigma(t)$ is stress behind the shock wave; σ_s is a yield stress; ρ is a density; E_p is a strain hardening modulus; m is a power of hardening. Initial conditions are:

$$t = 0: v = v_0, h = h_0$$

Conditions to determine a stopping time $t = t_f$ and the intact part's height h_f are:

$$v = 0, w = 0$$

One can transform the considered system of equations to:

$$dh / dt = -(v + w)$$

$$dv / dt = -\sigma_s / (\rho h)$$

$$w = \left(E_p / \rho \right)^{1/(m+1)} v^{(m-1)/(m+1)} - v$$

It seemed to be impossible to solve the system of equations for the arbitrary m . Therefore an approximate solution was made with the principal terms obtained at the point with $m = 1 + \gamma > 1$ where $\gamma \ll 1$. It was assumed the $\sigma - \varepsilon$ locus within a plastic domain close to linear ($E_p \ll E$ where E is a Young's modulus).

The result of the approximate solution is:

$$h(v) = h_0 \exp((v - v_0) / v_*)$$

$$v(t) = v_0 + v_* \ln(1 - t / t_0)$$

$$w(t) = v_c ((v / v_c)^{1/2} - v / v_c)$$

$$h_f = h_0 \exp(-v_0 / v_*)$$

$$t_f = t_0 (1 - \exp(-v_0 / v_*))$$

where $v_c = (E_p / \rho)^{1/2}$, $v_* = \sigma_s / (\rho E_p)^{1/2}$, $t_0 = h_0 / v_c$.

One can obtain the plastic zone's height h_p using the plastic wave speed $w(t)$:

$$h_p = \int_0^{t_f} w(t) dt = h_0 \left(\left(\frac{v_0}{v_c} \right)^{1/2} \left[1 - \exp\left(\frac{-v_0}{v_*} \right) \right] - \frac{v_0}{v_c} + \frac{v_*}{v_c} \left[1 - \exp\left(\frac{-v_0}{v_*} \right) \right] \right)$$

Total bar's height is $h_l = h_p + h_f$. It is important to notice that bar plastic part's shape is implicit according to the approach; therefore the shape is based on an assumption of incompressibility of bar's material.

Using the primary formulas for the numerical evaluation of the shape of the bar a number of initial conditions have been analyzed. Listed below parameters were used: particle's material is titanium; height $h_0 = 30 * 10^{-6}$ m; $r_0 = h_0 / 2$; density $\rho = 4500$ kg/m³; Young's modulus $E = 116 * 10^9$ Pa; Poisson's ratio $\nu = 0.32$; yield stress at normal conditions $\sigma_{s0} = 3 * 10^8$ Pa; initial velocity $v_0 = 20$ m/s. The bar's temperature is within the range $T = 1600 - 1660$ °C, whereas titanium's melting point T_m is 1665 °C. Defining yield stress at the temperature about the melting point is rather uncertain task; thus a linear approach $\sigma_s = \max(\sigma_{s0} (T_m - T) / T_m, 0)$ was used.

Fig. 2 and 3 shows the shape after collisions at different stream's temperatures, which are close to the melting point.

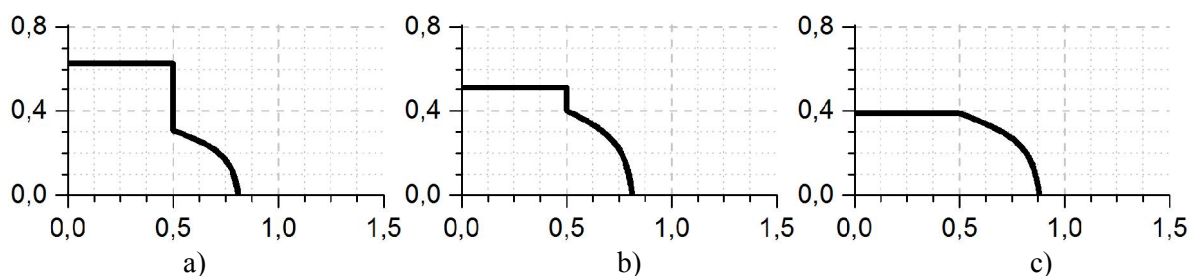


Figure 2. Particle's shape at $v_0 = 20$ m/s, $E_p = 2.5 * 10^7$ Pa, $m = 1.2$;

a) $T = 1600$; b) $T = 1620$; c) $T = 1633$.

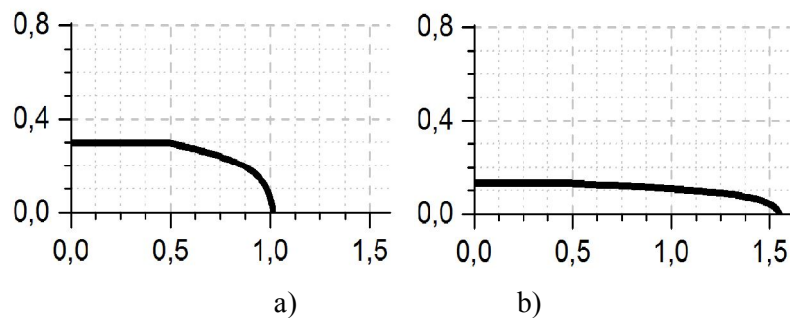


Figure 3. Particle's shape at $v_0 = 20$ m/s, $E_p = 2.5 * 10^7$ Pa, $m = 1.2$;
a) $T = 1640$; b) $T = 1650$.

Fig. 4 shows the shape after collisions at different stream's speeds. Fig. 5 shows an impact of the strain hardening modulus E_p and the power of hardening m on the shape.

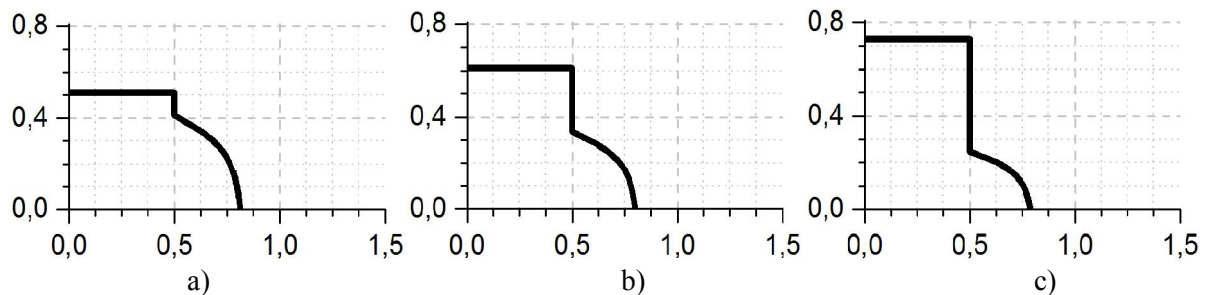


Figure 4. Particle's shape at $T = 1620$, $E_p = 2.5 * 10^7$ Pa, $m = 1.2$;
a) $v_0 = 20$ m/s; b) $v_0 = 15$ m/s, c) $v_0 = 10$ m/s.

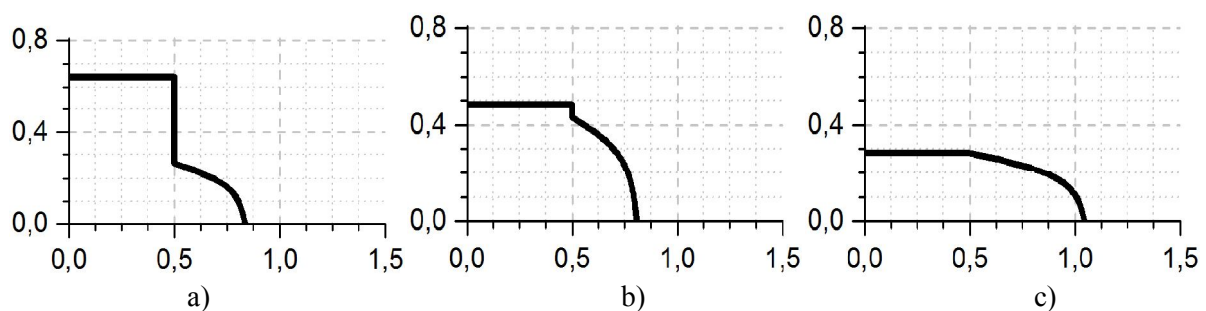


Figure 5. Particle's shape at $v_0 = 20$ m/s, $T = 1620$;
a) $E_p = 2.5 * 10^7$ Pa, $m = 1.8$; b) $E_p = 1 * 10^7$ Pa, $m = 1.8$; c) $E_p = 1 * 10^7$ Pa, $m = 1.2$.

3. Multidimensional modelling

In order to compare the above results a multidimensional modelling based on the numerical method from [5] was performed. Fig. 6 and 7 show ultimate bar's shapes with the same as Fig. 2 initial conditions.

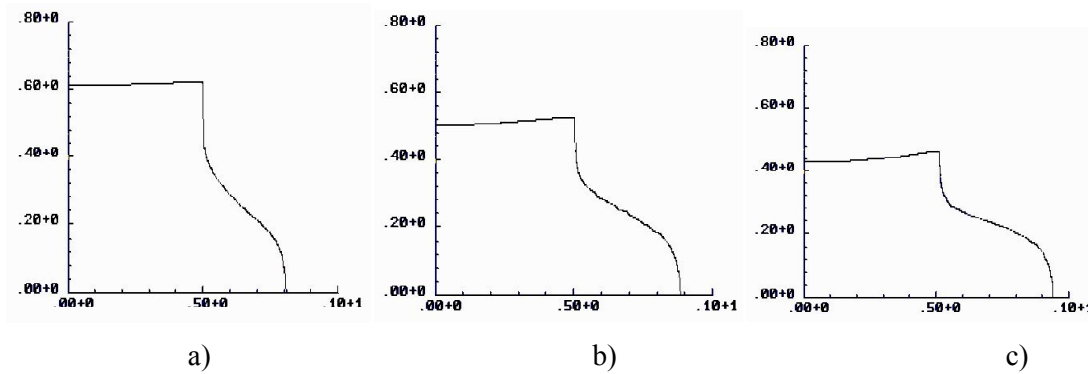


Figure 6. Particle's shape at $v_0 = 20$ m/s;
 a) $T = 1600$; b) $T = 1620$; c) $T = 1633$.

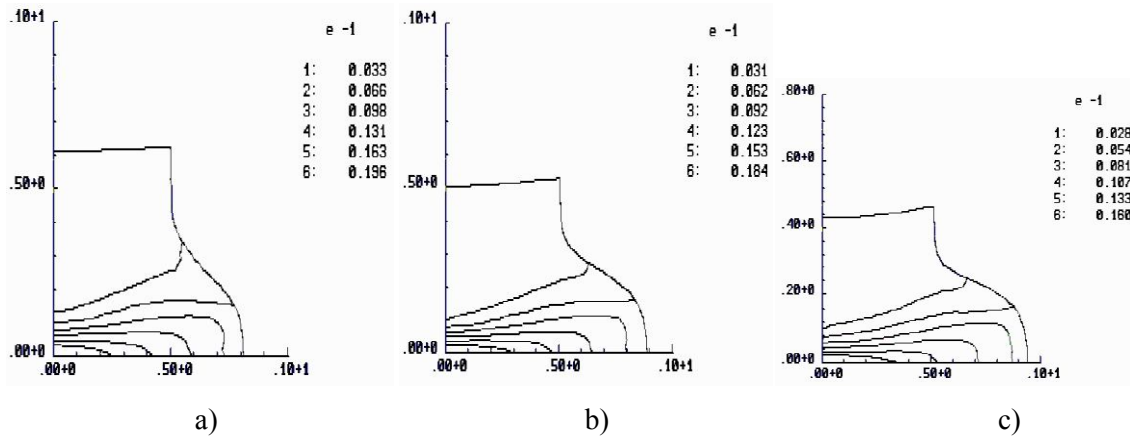


Figure 7. Contour plot of the plastic work at $v_0 = 20$ m/s;
 a) $T = 1600$; b) $T = 1620$; c) $T = 1633$.

Comparing Fig. 2 and Fig. 6 one can see the one-dimensional (bar-shaped) model gets ultimate thickness on the plastic part rather well within studied boundaries comparing to the multidimensional one, although it is important to notice bar's height and width are the same so this conclusion is not clear.

4. Conclusions

Approximate analytical estimates for the finite thickness of a deformed hot particle incident on an undeformed substrate, its radius and the collision time as a function of the speed are obtained for the law of hardening close to linear, by the perturbation method. The system of ordinary differential equations of the approximate model under consideration is numerically and analytically solved. A comparison of the numerical and analytical results in the one-dimensional approximation with the results of multivariate modelling of the unsteady collision process using finite-element schemes is carried out. The efficiency of the obtained formulas for estimating the thickness of the build-up layers as a function of particle velocity and flow temperature is shown.

References

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